Generalized Nonlinear Sigma Models and Universality in Three Dimensions. I. "Soft Mass" Renormalization of the $1/N$ Expansion

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Ultraviolet and infrared renormalized $1/N$ expansion of both high and low temperature phases as well as of the theory at the critical point of the $CP^{N-1}_{D=3}$ and $D=3$ Abelian—Higgs models with internal $SU(N)$ "flavour" symmetry is constructed. Generalized quantum chirality identities for composite operators are obtained from which the renormalizability of the CP^{N-1}_3 model follows. It is shown that the CP^{N-1}_3 model is an IR-stable fixed-point universal theory of the noncanonically renormalized Abelian Higgs model.

1. Introduction

Two-dimensional generalized nonlinear sigma models (CP^{N-1} [1] and supersymmetric extensions thereof [2], $G_{N,n}$ [3]) possess some most attractive features (instantons, asymptotic freedom, confinement), which make them a very good "laboratory" for modelling the complicated dynamics of realistic four-dimensional gauge theories. These models are in themselves very interesting also in higher (Euclidean) space-time dimensions $D>2$. In a recent note [4], it was demonstrated that the CP^{N-1}_3 model and the associated Abelian—Higgs model with internal $SU(N)$ "flavour" symmetry (AH_3) exhibit a second-order phase transition through the same mechanism as in the usual $O(N)$ (non) linear sigma ($(N)LS$) model [5, 6] (see also the Appendix). In the present paper we consider these models in the context of critical phenomena and universality [7]. Their (Euclidean) Lagrangians can be writren correspondingly in the following form

$$\mathscr{L}_{CP}(x)=-(\nabla_\nu\Phi)^*(\nabla_\nu\Phi)-\sigma'(\Phi^*\Phi-N_{\mu}/_T)-iNB\partial_\nu A_\nu, \tag{1}$$

$$\mathscr{L}_{AH}(x)=\mathscr{L}_{CP}(x)+N/2\lambda\mu^{\sigma'2}-N/4e^2\mu F^2_{\varkappa\nu}; \tag{1'}$$

$$\nabla_\nu=\partial_\nu+iA_\nu,\quad F_{\varkappa\nu}=\partial_\varkappa A_\nu-\partial_\nu A_\varkappa,\quad \Phi^*\Phi\equiv\sum_{c=1}^{N}\Phi^*_c\Phi_c,\ \text{etc.},$$

where λ, e^2 are dimensionless coupling constants, T—"temperature", μ – mass scale parameter; $\sigma'(x)=\sigma_0+i\sigma(x)$ (σ_0 is an arbitrary. but fixed non-negative constant) and $B(x)$ are auxiliary fields, the latter enforcing the Landau gauge. It will be more convenient in the sequel to introduce $u\equiv\lambda^{-1}$, $h\equiv e^{-2}$. The choice of Landau gauge is dictated by the necessity of ensuring optimal infrared behaviour of the $1/N$ expansion and scale invariance of the universal critical theory.

In section 2 the "soft mass" B o g o l i u b o v-P a r a s i u k-H e p p-Z i m m e r m a n n-L o w e n s t e i n (BPHZL) [8, 9] renormalization scheme is applied to construct a property ultraviolet (UV) and infrared (IR) renormalized (in each *separate* diagram) $1/N$ expansion of the CP^{N-1}_3 and AH_3 models allowing for a unified treatment of both high-temperature (HT) and low-temperature (LT) phases as well as of the critical theory (pre-asymptotic zero mass theory, respectively). In Section 3 the strict renormalizability of the $1/N$ expanded CP^{N-1}_3 model (which is nonrenormalizable with respect to the naïve T perturbation theory*)) is proved by means of the $SU(N)$ and $U(1)$-gauge Ward identities (WIs), the generalized quantum chirality identities (ChIs) (quantum analogues of the classical nonlinearity constraints) and Zimmermann's identities (ZIs) [10, 8] for composite operators. In Section 4 renormalization group (RG) and Callan-Symanzik (CS) equations (in both phases) are derived from which it is seen that the CP^{N-1}_3 model is an IR-stable fixed-point (u^*, $h^*=0$) theory of the VU noncanonically renormalized AH_3 model. More exactly, the $1/N$ expansion of the AH_3 model coinsides in this scaling limit with the one for the CP^{N-1}_3 model but additional logarithmic UV divergencies do arise in separate one-particle irreducible (IPI) subgraphs with six external Φ-lines. In a subsequent second part of the present work [11] it is proved that these divergencies nevertheless do cancel, due to the important ChIs. There the properties of the critical behaviour are also briefly analyzed and the scale invariance of the universal critical CP^{N-1}_3 theory is established by means of the explicitly constructed quantum Belinfante energy-momentum tensor.

The graphical rules of the $1/N$ expansion of the CP^{N-1}_3 and AH_3 models [4] are recalled in the Appendix. For the sake of compactness we shall employ the unified "soft-mass" notations (A.1-4) in order to treat simultaneously the CP^{N-1}_3 and AH_3 models in both phases and the theory at the critical point. All particular cases are obtained by substituting the concrete values of the parameters m, f, u, h.

Finally, we note that the whole treatment here and that in [11] closely resembles the one presented in [12, 13] for the $O(N)$ $(\varphi^4)_3$ and the usual $D=3$ $O(N)$ NLS models.

2. Construction of the BPHZL Renormalized $1/N$ Expansion

The systematic BPHZL renormalization scheme is founded on the basic concepts of UV ($d(\gamma)$, $\delta(\gamma)$) and IR ($r(\gamma)$, $\varrho(\gamma)$) degrees of IPI (sub) graphs (γ) and on (the modified) Z i m m e r m a n n's "forest formula" [8]. In the present context the latter attributes to each (connected) $1/N$ graph $G\equiv G^{\{P_a\}}_{\{L_\Phi, L_\sigma, L_A\}}$ of (connected) Green's functions of the general type ($\delta_a\geq d_{P_a}$, $\varrho_a\leq r_{P_a}$; $d_{P_a}(r_{P_a})$ — canonical UV (IR) dimensions of the composite operators $P_a\equiv P_a(\Phi, \sigma, A_\mu)$

$$\langle\prod_a \mathfrak{N}^{\varrho_a}_{\delta_a}[P_a](x_a)X\rangle,\quad X\equiv\prod_{i=1}^{L_\Phi}\Phi_{c_i}(x'_i)\prod_{j=1}^{L_\sigma}\sigma(x''_j)\prod_{k=1}^{L_A}A_{\nu_k}(x'''_k) \tag{2}$$

*) We need not address the highly IR singular double T and $\varepsilon=D-2$ expansion [5].

a well-defined expression

$$(3)\qquad \mathcal{R}_G((p)) = \int \prod \frac{d^3K}{(2\pi)^3} R_G((p),(k))\,; \quad R_G = \sum_{U \in F_G} \prod_{\gamma \in U} (-\tau_\gamma) I_G.$$

In (3) all notations are standard: F_G — set of all G — forests U; R_G, I_G — renormalized, unrenormalized, respectively, integrand; (p), (k) — sets of external, internal respectively, momenta;

$$(4)\qquad \tau_\gamma \equiv \tau_\gamma^{\delta(\gamma),\,\varrho(\gamma)} = t^{\varrho(\gamma)-1}_{p,s-1} + (1 - t^{\varrho(\gamma)-1}_{p,s-1}) t^{\delta(\gamma)}_{p,s},$$

$t^d_{x,y}$ being the standard Taylor subtraction operator of order d in the variables x, y ($\tau_\gamma \equiv 0$ for 1PR γ). After implementing all subtractions one sets $s=1$.

Now the main point is to specify the subtraction degrees $\delta(\gamma)$, $\varrho(\gamma)$ (we shall need in what follows explicitly only the case δ_a, $\varrho_a \leq 3$ in (2)). $\delta(\gamma)$, $\varrho(\gamma)$ are subject to the consistency constraints [9, 14]

$$(5)\qquad \begin{aligned} &\delta(\gamma) \geq d(\gamma), \quad \delta(\gamma) \geq d(\gamma/\{\lambda\}) + \sum_{i=1}^{c} \delta(\lambda_i), \\ &\varrho(\gamma) \leq r(\gamma), \quad \varrho(\gamma) \leq r(\gamma/\{\lambda\}) + \sum_{i=1}^{c} \varrho(\lambda_i), \end{aligned}$$

where $\{\lambda\} \equiv \{\lambda_1, \ldots, \lambda_c\}$ is an arbitrary set of mutually disjoint 1PI subgraphs of γ and $\gamma/\{\lambda\}$ denotes the corresponding reduced graph (i. e. all λ_i contracted to points). $\delta(\gamma)$, $\varrho(\gamma)$ should satisfy the IR convergence criterium [8] ($\hat{G}$ is the so called augmented graph, i. e. G complemented by auxiliary lines beginning at the external vertices of G and ending at a common new vertex V_0)

$$(6)\qquad r(\widehat{G}/\{\lambda\}) + \sum_{i=1}^{c} \max\{0, \varrho(\lambda_i)\} > 0.$$

The canonical degrees $d(\gamma)$, $r(\gamma)$ are determined starting from the general definitions of Ref. [14] and accounting for the explicit form of the $1/N$ graphical rules (A. 1—4)

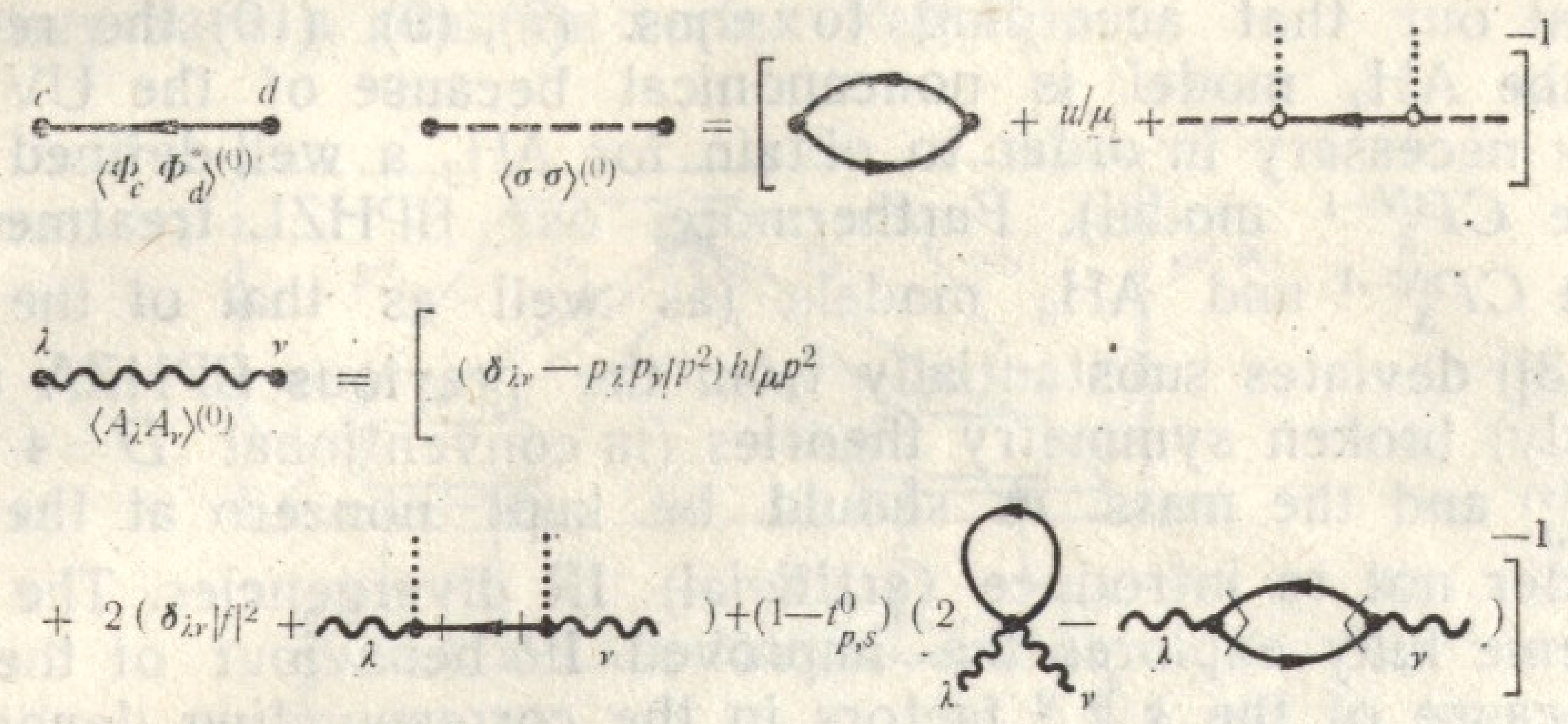
$$(7)\qquad \begin{aligned} &d_{CP}(\gamma) = 3 - 1/2 L_\Phi(\gamma) - 2L_\sigma(\gamma) - L_A(\gamma) + \sum_{V(P_a) \in \gamma} (d_{P_a} - 3), \\ &d_{AH}(\gamma) = d_{CP}(\gamma) - (\mathcal{L}_\sigma(\gamma) + \mathcal{L}_A(\gamma))\,; \end{aligned}$$

$$(8)\qquad \begin{aligned} &r_{cr}(\gamma) = 3 - 1/2 L_\Phi(\gamma) - 2L_\sigma(\gamma) - L_A(\gamma) + \sum_{V(P_a) \in \gamma} (r_{P_a} - 3), \quad (T = T_c), \\ &r_{HT}(\gamma) = r_{cr}(\gamma) + (2\mathcal{L}_\Phi(\gamma) - \mathcal{L}_\sigma(\gamma) - \mathcal{L}_A(\gamma)), \\ &r_{LT}(\gamma) = r_{cr}(\gamma) + (\mathcal{L}_\sigma(\gamma) + \mathcal{L}_A(\gamma) - 1/2 L_F(\gamma)). \end{aligned}$$

Here $L_\psi(\gamma)$ ($\mathcal{L}_\psi(\gamma)$) denote the number of external (internal) Φ_c, σ, A_ν, Φ_F — lines of $\gamma \equiv \gamma^{\{P_a\}}_{\{L_\Phi, L_\sigma, L_A;\, L_{\Phi_F}\}}$ respectively; $L_F(\gamma)$ — number of external "blind" F-lines; $V(P_a)$ — vertices of P_a.

A careful analysis of the UV and IR behaviours as given by (7), (8), (5) (6) leads to the following choice*)

$$(9)\qquad \delta_{CP(AH)}(\gamma) = 3 - 1/2[L_\Phi(\gamma) - L^{t.p.}_\Phi(\gamma)] - 2[L_\sigma(\gamma) - L^{t.p.}_\sigma(\gamma)] - L_A(\gamma) + \sum_{V(P_a) \in \gamma} (\delta_a - 3)$$



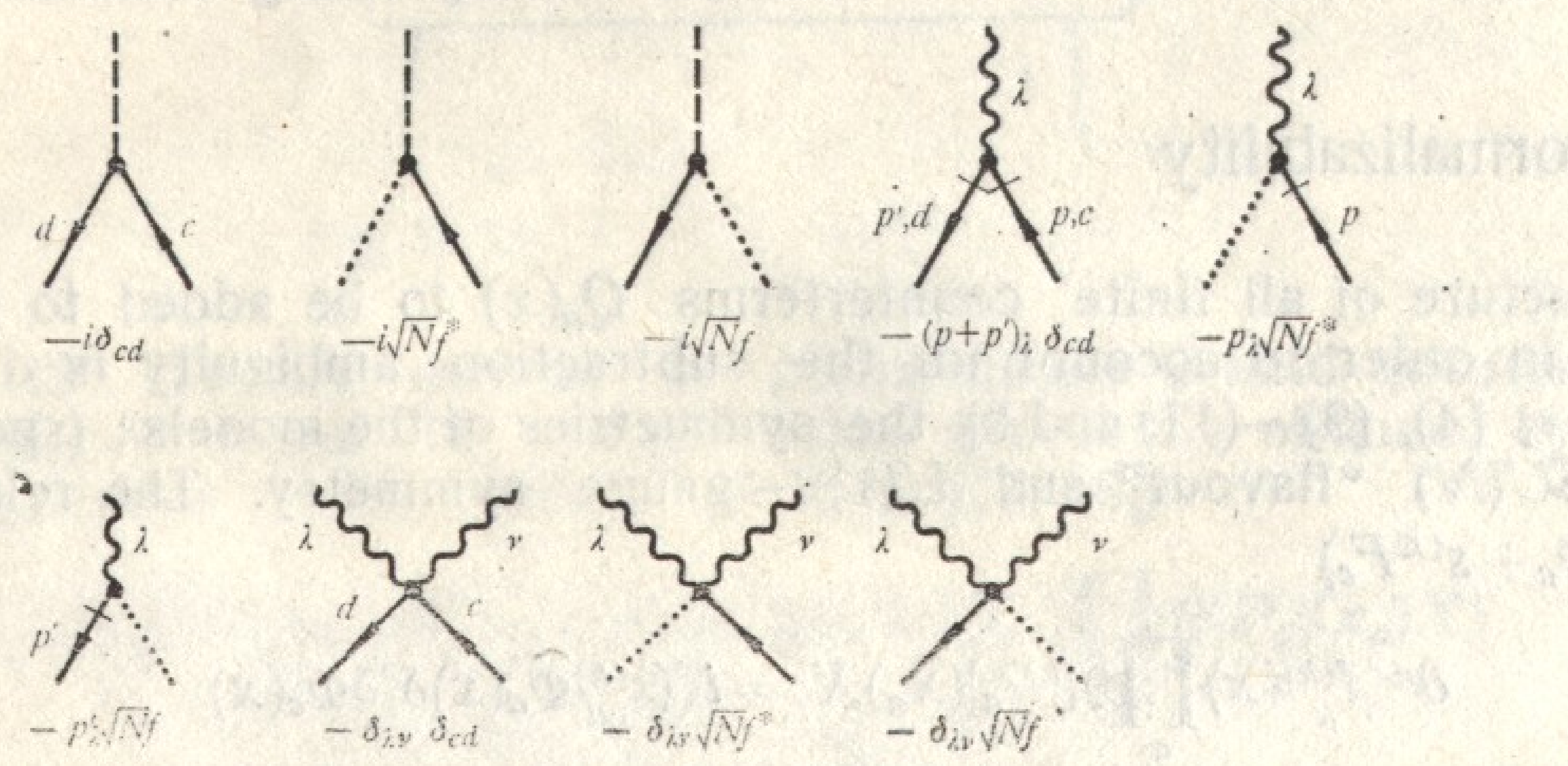


Fig. 1

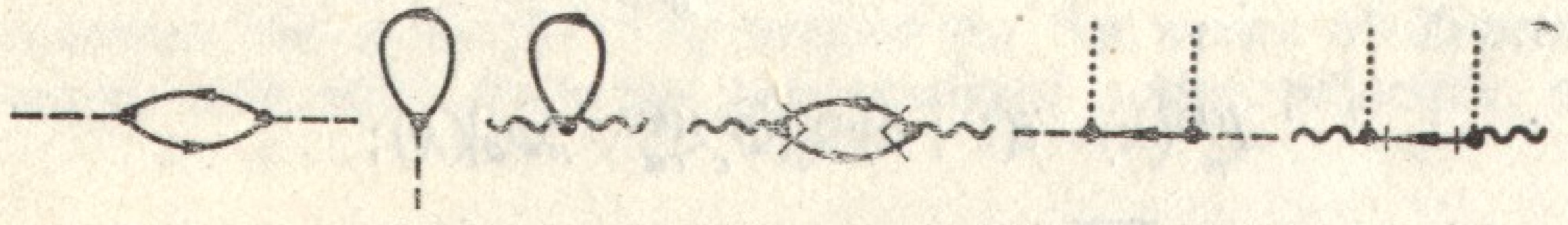


Fig. 2

except for $\dot{\gamma}$ with $L_\sigma = L_A = 0$; $(L_\Phi - L^{t.p.}_\Phi, L_{\Phi_F}) = (6,0), (5,1), (4,2), (3,3)$

$$(10)\qquad \delta_{AH}(\dot{\gamma}) = \delta_{CP}(\dot{\gamma}) - 1 - 1/2 L_{\Phi_F}(\dot{\gamma}) < 0\,;$$

$$(11)\qquad \varrho_{CP(AH)}(\gamma) = \delta_{CP(AH)}(\gamma) + \sum_{(P_a) \in \gamma} (\varrho_a - \delta_a) \begin{cases} +0 & \text{(LT phase, } T = T_c \text{ theory)} \\ -2 \text{ for } \gamma = \gamma_{(4,0,0)} & \\ +1 \text{ for } \gamma = \gamma_{(0,0,2)} & \text{(HT phase),} \\ -1 \text{ for } \gamma = \gamma_{(2,0,0)} & \\ +0 \text{ for the remaining } \gamma & \end{cases}$$

*) Subgraphs γ with $L_F(\gamma) + L^{t.p.}_\Phi(\gamma)$ odd (in the LT phase) have half-integer $\delta(\gamma)$, $\varrho(\gamma)$ according to (9)—(11) and contain at the same time factors $s^{1/2 L_F(\gamma)}$. Then, by definition (cf. [12, 13]):

$$\tau^{\delta(\gamma),\,\varrho(\gamma)} s^{1/2}(\ldots) = s^{1/2} \tau^{\delta(\gamma)-1/2,\,\varrho(\gamma)-1/2}(\ldots).$$

where $L^{t.p.}_{\Phi,\sigma}(\gamma)$ denote the number of external Φ, σ-lines carring Φ_F, σ-tadpoles ($L^{t.p.}_{\Phi(\sigma)}(\gamma) \neq 0$ only in the LT(HT) phase, respectively). This means that Φ_F-tadpoles are treated on an equal footing as the external F-lines (cf. [12]).

The proof of the UV and IR absolute convergence of $\mathcal{R}_G((p))$ (3) closely follows the pattern of Ref. [9] (cf. [12] for the case of the O(N) NLS model).

We point out that according to eqns. (7), (9), (10) the renormalization scheme for the AH_3 model is noncanonical because of the UV oversubtractions. This is necessary in order to obtain for AH_3 a well-defined scaling limit (which is the CP^{N-1}_3 model). Furthermore, our BPHZL treatment of the LT phase of the CP^{N-1}_3 and AH_3 models (as well as that of the O(N) (N)LS model [12, 13]) deviates substantially from the previous BPHZA treatments of (spontaneously) broken symmetry theories (in conventional $D=4$ models) [15], where $\tau_\gamma = t^{\delta(\gamma)}_{p,s}$ and the mass m should be kept nonzero at the intermediate stages in order not to introduce (artificial) IR divergencies. The present subtraction scheme fully explores the improved IR behaviour of the propagators (A. 2—3) because of the $s|f|^2$ factors in the corresponding denominators.

3. Renormalizability

The structure of all finite counterterms $Q_a(x)$ to be added to (1) or (1′) respectively in order to account for the subtraction ambiguity is dictated by the form of τ_γ (4), (9)—(11) and by the symmetries of the models: (spontaneously broken) $SU(N)$ "flavour" and $U(1)$ — gauge symmetry. The relevant WIs read ($\widehat{\Phi}_c \equiv \Phi_c + s^{1/2} F_c$)

$$\partial^\mu \langle I^{(k)}_\mu(x) \prod_a \mathfrak{N}[P_a](x_a) X \rangle = i \langle (\lambda^{(k)}_{cd} \widehat{\Phi}_d(x) \delta/\delta\Phi_c(x) + (\lambda^{(k)*})_{cd} \widehat{\Phi}^*_d(x) \delta/\delta\Phi^*_c(x)) \prod_a \mathfrak{N}[P_a](x_a) X \rangle, \tag{12}$$

$$I^{(k)}_\mu(x) = i(1+b)\mathfrak{N}^2_2[\widehat{\Phi}^*_c \lambda^{(k)}_{cd} \overleftrightarrow{\nabla}_\mu \widehat{\Phi}_d](x); \tag{13}$$

$$\partial^\mu \langle J_\mu(x) \prod_a \mathfrak{N}[P_a](x_a) X \rangle = i \langle (\widehat{\Phi}_c(x) \delta/\delta\Phi_c(x) - \widehat{\Phi}^*_c(x) \delta/\delta\Phi^*_c(x)) \prod_a \mathfrak{N}[P_a](x_a) X \rangle, \tag{14}$$

$$J_\mu(x) = i(1+b)\mathfrak{N}^2_2[\widehat{\Phi}^*_c \overleftrightarrow{\nabla}_\mu \widehat{\Phi}_c](x), \tag{15}$$

where $\lambda^{(k)}$, k=1, . . ., N^2-1 form a SU(N) generator basis and b turns out to be the coefficient of the gauge invariant Φ-field renormalization counterterm (see (24) below). In (12), (14) the following notation is used

$$\mathfrak{N}^\varrho_\delta[P\delta/\delta\psi](x)\mathfrak{N}^{\varrho'}_{\delta'}[Q](y) \equiv \sum_n \sum_{k=0}^{n} (-1)^k \binom{n}{k} \partial^k_{(\mu)} \delta(x-y)$$
$$\times \mathfrak{N}^{\varrho+\varrho'-k-3}_{\delta+\delta'-k-3}[\partial Q/\partial(\partial^k_{(\mu)}\psi) \mid \partial_{\mu_{k+1}} \ldots \partial_{\mu_n} P](y), \quad \partial^k_{(\mu)} \equiv \prod_{i=1}^{k} \partial_{\mu_i}.$$

Here an example of a special type of anisotropic normal products [16] does appear: the so called $P_1|P_2$ factorized normal products $\mathfrak{N}[P_1|P_2]$ [12]. The latter differ from the usual (isotropic) one $\mathfrak{N}[P_1P_2]$ in that for each factorized subgraph $\gamma^{P_1P_2} = \gamma^{P_1} \bigcup \gamma^{P_2}$, $\gamma^{P_1} \bigcap \gamma^{P_2} = \{V(P_1P_2)\}$ (see Fig. 3) γ^{P_1} and γ^{P_2} are treated as **non**-overlapping in the corresponding "forest formula" (3) (inspite of the fact that they have a common vertex $V(P_1P_2)$).

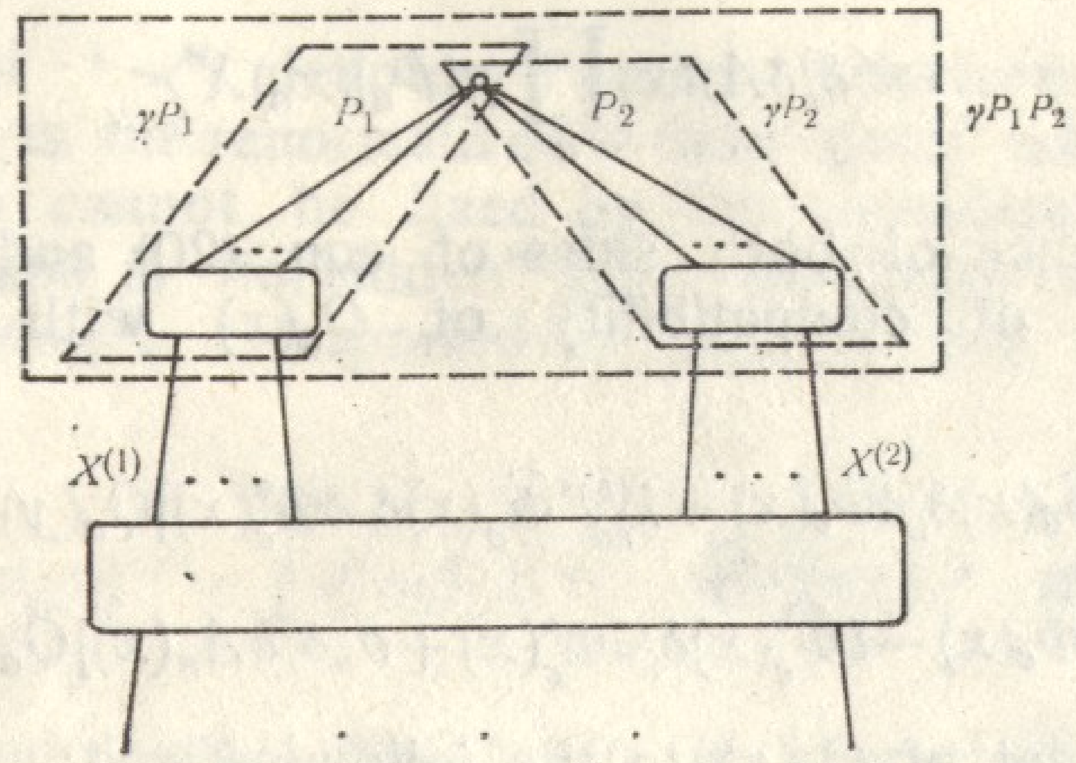


Fig. 3

Eqns. (12), (14) are derived by means of the Φ-field quantum equations of motion (QEM) (eqns. (16) and (20) below are easily obtained by a straightforward application of the methods of Refs. [17, 16]):

$$\langle \mathfrak{N}^\varrho_\delta[P/\partial\mathcal{L}_{\text{eff}}/\partial\Phi^*_c - \partial_\mu(\partial\mathcal{L}_{\text{eff}}/\partial(\partial_\mu\Phi^*_c))](x) \prod_a \mathfrak{N}[P_a](x_a) X \rangle = -\langle \mathfrak{N}^\varrho_\delta[P\delta/\delta\Phi^*_c](x) \prod_a \mathfrak{N}[P_a](x_a) X \rangle, \tag{16}$$

where $\mathcal{L}_{\text{eff}}$ denotes the effective Lagrangian (in the sense of Zimmermann [10] or, equivalently, with respect to the renormalized action principle, see the next Section)

$$\mathcal{L}^{\text{eff}}_{CP}(x) = -\mathfrak{N}^3_3[(\nabla_\nu\widehat{\Phi}_c)^*(\nabla_\nu\widehat{\Phi}_c) + m^2(s)\Phi^*_c\Phi_c + i\sigma(\Phi^*_c\Phi_c + s^{1/2}F^*_c\Phi_c + s^{1/2}\Phi^*_c F_c)$$
$$+ \mu/2(1-s)A_\nu A_\nu + iNB\partial_\nu A_\nu]_{\otimes}(x) + iNa\sigma(x) + \sum_a c_a \mathfrak{N}^3_3[Q_a]_{\otimes}(x); \tag{17}$$

$$\mathcal{L}^{\text{eff}}_{AH}(x) = \mathcal{L}^{\text{eff}\prime}_{CP}(x) - N/2\mu\mathfrak{N}^3_3[u\sigma^2 + h/2F^2_{\varkappa\lambda}]_{\otimes}(x). \tag{18}$$

The subscript "$\otimes$" means omission of vacuum graphs. The prime in $\mathcal{L}^{\text{eff}\prime}_{CP}(x)$ indicates that the counterterm $\mathfrak{N}^3_3[(\Phi^*\Phi + s^{1/2}F^*\Phi + s^{1/2}\Phi^*F)^3]_{\otimes}$ has to be dropped for the AH_3 model (recall (10)).

The local $U(1)$ gauge invariance WI ($\Delta = \partial_\lambda\partial_\lambda$):

$$-iN\langle \Delta B(x) \prod_a \mathfrak{N}[P_a](x_a) X \rangle = \langle (i\widehat{\Phi}_c(x)\delta/\delta\Phi_c(x) - i\widehat{\Phi}^*_c(x)\delta/\delta\Phi^*_c(x) + \partial_\mu\delta/\delta A_\mu(x)) \prod_a \mathfrak{N}[P_a](x_a) X \rangle \tag{19}$$

must follow from the QEM for A_μ:

$$\langle[\partial_\mu(\partial\mathcal{L}_{\text{eff}}/\partial(\partial_\mu A_\lambda)-\partial\mathcal{L}_{\text{eff}}/\partial A_\lambda](x)\prod_a\mathfrak{N}[P_a](x_a)X\rangle$$

$$\equiv\langle(N/\mu h\partial_\nu F_{\lambda\nu}-iN\partial_\lambda B-J_\lambda)(x)\prod_a\mathfrak{N}[P_a](x_a)X\rangle \tag{20}$$

$$=\langle\delta/\delta A_\lambda(x)\prod_a\mathfrak{N}[P_a](x_a)X\rangle$$

by taking the divergence of both sides of eqn. (20) and accounting for (14).

The requirement of compatibility of $Q_a(x)$ with the WIs (12), (14) and (19)

$$[\lambda^{(k)}_{cd}\widehat{\Phi}_d(x)\delta/\delta\Phi_c(x)+\lambda^{(k)*}_{cd}\widehat{\Phi}^*_d(x)\delta/\delta\Phi^*_c(x)]Q_a(y)=0,$$

$$[i\widehat{\Phi}_c(x)\delta/\delta\Phi_c(x)-i\widehat{\Phi}^*_c(x)\delta/\delta\Phi^*_c(x)+\partial_\mu\delta/\delta A_\mu(x)]Q_a(y)=0,$$

restricts the possible set of $Q_a(x)$ to the following one:

$$Q_0\equiv i\sigma(x),\ Q_1=-(\nabla_\nu\widehat{\Phi}_c)^*(\nabla_\nu\widehat{\Phi}_c),\ Q_2=-i\sigma(\Phi^*_c\Phi_c+s^{1/2}F^*_c\Phi_c+s^{1/2}\Phi^*_cF_c),$$

$$Q_3=-(\Phi^*_c\Phi_c+s^{1/2}F^*_c\Phi_c+s^{1/2}\Phi^*_cF_c),\ Q_4=-(1-s)\Phi^*_c\Phi_c,\ Q_5=-(1-s)^2\Phi^*_c\Phi_c, \tag{21}$$

$$Q_6=-(Q_3)^2,\quad Q_7=(1-s)Q_6,\quad Q_8=-1/2(1-s)A_\nu A_\nu,\quad Q_9=(Q_3)^3.$$

Now two fundamental types of algebraic identities among composite operators will play a crucial role: the ZIs [10] (which are typical for all BPHZ renormalized models) and the generalized quantum ChIs. The latter are specific for quantum models in which classical versions the fields assume values on nonlinear manifolds [12]. Let us emphasize that just like for the O(N) NLS model all components of the quantum CP^{N-1} field $\Phi_c(x)$, $c=1,\dots,N$ are independent. The ChIs can also be viewed as QEM for the auxiliary σ-field and thus they are valid up to additional terms vanishing in the scaling limit (the first term on the r. h. s. of eq. (22)) also in the AH_3 model. In the last case we shall call them prechirality identities (PChIs).

The ZIs enable us to express all (oversubtracted) $\mathfrak{N}^3_3[Q_i]_\otimes$, $i=3,\dots,8$ in terms of canonical ones Q_0, $\mathfrak{N}^3_3[Q_j]_\otimes$, $j=1,2,9$ (the latter enters only in the case of the CP^{N-1}_3 model) and also of $\mathfrak{N}^2_2[Q_6]_\otimes$, $\mathfrak{N}^1_1[Q_3]_\otimes$, $\mathfrak{N}^3_3[\sigma^2]_\otimes$ (the last one appears only in the AH_3 model) by a well-known techniques [8, 14].

The (P) ChIs have the following general form

$$\langle\mathfrak{N}^\varrho_\delta[P(\Phi^*\Phi+s^{1/2}F^*\Phi+s^{1/2}\Phi^*F)](x)X\rangle=iNu/\mu\langle\mathfrak{N}^\varrho_\delta[\{P\}\sigma](x)X\rangle$$

$$+N([\mathfrak{D}(0;m(s))]^{ren}+a)\langle\mathfrak{N}^{\varrho-1}_{\delta-1}[P](x)X\rangle-i\sum_{j=1}^{L_\sigma}\delta(x-x''_j) \tag{22}$$

$$\times\langle\mathfrak{N}^{\varrho-1}_{\delta-1}[P](x)\partial/\partial\sigma(x''_j)X\rangle-c_2\langle\mathfrak{N}^\varrho_\delta[P\,|\,(\widehat{\Phi}^*\widehat{\Phi}-s\mathfrak{N}\,|f\,|^2)](x)X\rangle$$

$$+\sum_{X^{(1)}X^{(2)}}\langle A_{X^{(1)}X^{(2)}}(x)X\rangle;$$

$$A_{X^{(1)}X^{(2)}}(x)\equiv-\mathfrak{N}^\varrho_\delta[\tau^{\delta_A,\varrho_A}(\langle\mathfrak{N}^{\varrho-1}_{\delta-1}[P](0)\widetilde{X}^{(1)}\rangle^{1PI}\langle\sigma(0)\widetilde{X}^{(2)}\rangle^{1PI})$$

$$\times X^{(1)}X^{(2)}](x),$$

$$\left.\begin{matrix}\delta_A\\ \varrho_A\end{matrix}\right\}=\left.\begin{matrix}\delta\\ \varrho\end{matrix}\right\}=1/2(L^{(1)}_\Phi+L^{(2)}_\Phi)-2(L^{(1)}_\sigma+L^{(2)}_\sigma)-(L^{(1)}_A+L^{(2)}_A),\quad(L^{(i)}_\psi\equiv L_\psi(X^{(i)})).$$

Here the superscript "~" denotes Fourier transform; $[\mathfrak{D}(0;\ m)]^{ren}(2\pi)^3\equiv[\int d^3k(k^2+m^2)^{-1}]^{ren}$ is the renormalized Φ-field point loop. The corresponding subtraction ambiguity cannot be fixed by the normalization conditions (25)—(27) since this subgraph is "forbidden" for usual Green's functions (see Fig. 2). For $\mathfrak{N}^3_3[Q_a]_\otimes$ $\mathfrak{D}(0;m(s))$ is interpreted as $\gamma_{(0,1,0;0)}$ and according to (9), (11) we have

$$[\mathfrak{D}(0;m(s))]^{ren}=\int d^3k(2\pi)^{-3}(1-\tau^{1,1})[m(s)^2+k^2]^{-1}=0.$$

The anisotropic normal product $\mathfrak{N}^\varrho_\delta[\{P\}\sigma](x)$ is characterized by the property that to each subgraph containing lines belonging to P one oversubtraction is assigned. The derivation of eqns. (22) closely parallels that for the O(N) (N)LS model [12]. The last sum on the r. h. s. of (22) is entirely due to IR renormalization effects and has no classical analogue.

Let us note that the relevant Green's functions (in terms of which the "big" Hilbert space is constructed) involve X with $L_\sigma=0$ only (in the notations of (2)) since the auxiliary σ-field does not correspond to any particle. Bearing this in mind we deduce from (22) for the CP^{N-1}_3.

$$\mathfrak{N}^1_1[\Phi^*\Phi+s^{1/2}F^*\Phi+s^{1/2}\Phi^*F](x)=Na=\underbrace{\text{const}}.\ 1, \tag{23.a}$$

$$\mathfrak{N}^2_2[(\Phi^*\Phi+s^{1/2}\Phi^*F+s^{1/2}F^*\Phi)^2](x)=\underbrace{\text{const}}.\ 1+a\sigma(x), \tag{23.b}$$

$$\mathfrak{N}^3_3[Q_9]_\otimes(x)=a_0\sigma(x)+a_1\mathfrak{N}^3_3[Q_1]_\otimes(x), \tag{23.c}$$

$$\mathfrak{N}^3_3[Q_2]_\otimes(x)=Na(1+c_2)^{-1}\sigma(x), \tag{23.d}$$

where a, a_0, a_1 are simply expressed in terms of 1PI Green's functions. In (23. *b*, *c*) the WI (19) and the ZIs were also used. In the AH_3 model (and or general X) we obtain instead of (23.*d*)

$$(1+c_2)\langle\mathfrak{N}^3_3[Q_2]_\otimes(x)X\rangle=Nu/\mu\langle\mathfrak{N}^3_3[\sigma^2]_\otimes(x)X\rangle \tag{23.e}$$

$$-iNa\langle\sigma(x)X\rangle-\langle X\rangle\sum_{j=1}^{L_\sigma}\delta(x-x''_j).$$

Eqns. (23. *a*, *b*) (and their analogues for the AH_3 model) and eqns. (23. *c*, *d*, *e*) complete the proof of the renormalizability of the BPHZL renormalized $1/N$ expanded CP^{N-1}_3 and AH_3 models, i. e. the only independent counterterms (21) are Q_i, $i=0,1$ (for CP^{N-1}_3) and Q_i, $i=0,1,2$ (for AH_3), respectively. The final form of $\mathcal{L}_{\text{eff}}$ (17) reads ($c_1\equiv b$, $c_2\equiv c$)

$$\mathcal{L}_{\mathrm{eff}}(x) = -\mathfrak{N}_3^3[(1+b)(\nabla_\nu\widehat{\Phi})^*(\nabla_\nu\widehat{\Phi}) + m^2(s)\Phi^*\Phi$$

$$+(1+c)i\sigma(\Phi^*\Phi + s^{1/2}F^*\Phi + s^{1/2}\Phi^*F) + \mu/2(1-s)A_\lambda A_\lambda + iNB\partial_\lambda A_\lambda \tag{24}$$

$$+N/_{2\mu}u\sigma^2 + N/_{4u}hF^2_{\lambda\nu}]_{\otimes}(x) + iNa\sigma(x).$$

For the CP_3^{N-1} model $u=h=c\equiv 0$.

The coefficients a, b, c are determined by means of the "physical" normalization conditions. In the HT phase ($a \equiv m\widetilde{a}(m/\mu, u, h)$

$$\Gamma_{cd}^{(2,0,0)}|_{p^2=-m^2} \equiv \langle \widetilde{\Phi}_c(p)\widetilde{\Phi}_d^*(-p)\rangle^{1PI}|_{p^2=-m^2} = 0,\ \Gamma_{cd}^{(2,0,0)}|_{p^2=\mu^2} = -\delta_{cd}(\mu^2+m^2),$$

$$\Gamma_{cd}^{(2,1,0)}{}_{s.p.\mu^2} \equiv \langle \sigma(0)\widetilde{\Phi}_c(p)\widetilde{\Phi}_d^*(q)\rangle^{1PI}|_{p^2=q^2=pq=\mu^2} = -i\delta_{cd}. \tag{25}$$

In the LT phase $a \equiv |f|^2\widetilde{a}(|f|^2/\mu, u, h) = 0$ (this fact follows from the first normalization condition (26) in the same manner as in the O(N) (N)LS model[12])

$$\langle\Phi_c\rangle = 0;\ \Gamma_\perp^{(2,0,0)} \equiv (\delta_{cd} - \widehat{F}_c\widehat{F}_d^*)\Gamma_{db}^{(2,0,0)}|_{p^2=\mu^2} = -\mu^2(\delta_{cb} - \widehat{F}_c\widehat{F}_b^*),$$

$$\Gamma_\perp^{(2,1,0)} \equiv (\delta_{cd} - \widehat{F}_c\widehat{F}_d^*)\Gamma_{cb}^{(2,1,0)}|_{s.p.\mu^2} = -i(\delta_{cb} - \widehat{F}_c\widehat{F}_c^*). \tag{26}$$

In the $T=T_c$ theory there is no $iNa\sigma(x)$ counterterm (this ensures the masslessness) and we are left with

$$\Gamma_{cd}^{(2,0,0)}{}_{p^2=\mu^2} = -\mu^2\delta_{cd},\quad \Gamma_{cd}^{(2,1,0)}|_{s.p.\mu^2} = -i\delta_{cd}. \tag{27}$$

4. Partial Differential Equations for the Green Functions

RG equations and (broken) dilatation WIs (CS eqns. in the HT phase and their analogues in the LT phase) can be derived in the present context with the aid of the well-known method of differential-vertex operations [18].

The renormalized action principle (which can be proved in the present $1/N$ framework exactly along the lines of the corresponding proof for the O(N) (N)LS model [12]) gives (see (24))

$$\xi\partial/\partial\xi\langle X\rangle = \langle \xi\partial S_{\mathrm{eff}}/\partial\xi X\rangle,\quad \xi = \mu, m, f^{(*)}, u, h; \tag{28}$$

$$S_{\mathrm{eff}} = \int d^3x\mathcal{L}_{\mathrm{eff}}(x) \equiv Na\Delta_0 + (1+b)\Delta_1 + (1+c)\Delta_2 + m^2\Delta_3$$

$$+2m(\mu-m)\Delta_4 + (\mu-m)^2\Delta_5 + \mu\Delta_8 + Nu/\mu\Delta_{\sigma\sigma} + Nh/\mu\Delta_F$$

$$-i\int d^3x\mathfrak{N}_3^3[B\partial_\nu A_\nu]_{\otimes}(x);\quad \Delta_i \equiv \int d^3x\mathfrak{N}_3^3[Q_i]_{\otimes}(x),$$

$$\Delta_{\sigma\sigma} \equiv -1/2\int d^3x\mathfrak{N}_3^3[\sigma^2]_{\otimes}(x),\quad \Delta_F \equiv -1/4\int d^3x\mathfrak{N}_3^3[F^2_{\lambda\nu}]_{\otimes}(x).$$

Explicitly for $\xi = f^{(*)}$ eqn. (28) reads

$$\{f\partial/\partial f + f^*\partial/\partial f^* - 2|f|^2(\partial a/\partial|f|^2\Delta_0 + \Delta b/\partial|f|^2\Delta_1 + \partial c/\partial|f|^2\Delta_2)$$

$$-\widetilde{\Delta}\}\langle X\rangle = 0,$$

$$\widetilde{\Delta} \equiv -(1+b)\int d^3x\mathfrak{N}_3^3[A_\nu A_\nu(s^{1/2}F^*\Phi + s^{1/2}\Phi^*F + 2Ns|f|^2)]_{\otimes}(x)$$

$$-i(1+c)\int d^3x\mathfrak{N}_3^3[\sigma(\Phi^*F + F^*\Phi)s^{1/2}]_{\otimes}(x).$$

Furthermore, QEM (16) yields

$$\{(1+b)\Delta_1 + (1+c)\Delta_2 + m^2\Delta_3 + 2m(\mu-m)\Delta_4 + (\mu-m)^2\Delta_5$$

$$+1/2\widetilde{\Delta} + 1/2L_\Phi\}\langle X\rangle = 0. \tag{29}$$

Finally, we use the (gauge invariant) ZIs

$$\Delta_j = \begin{cases} r_{j0}\Delta_0 + r_{j1}\Delta_1 & CP_3^{N-1} \\ r_{j0}\Delta_0 + r_{j1}\Delta_1 + r_{j2}\Delta_2 & \mathrm{AH}_3 \end{cases} \qquad j = 3, 4, 5, 8$$

and the (gauge invariant) integrated (P) ChIs (23.*e*)

$$\{(1+c)\Delta_2 + Na\Delta_0 + 2Nu/\mu\Delta_{\sigma\sigma} + L_\sigma\}\langle X\rangle = 0$$

to express the composite vertex insertions in eqns. (28), (29) (four equations for CP_3^{N-1}, six equations for AH_3, respectively) in terms of the independent ones: $\Delta_0, \Delta_1, \widetilde{\Delta}$ (for CP_3^{N-1}); $\Delta_0, \Delta_1, \widetilde{\Delta}, \Delta_2, \Delta_F$ (for AH_3). Δ_0 gives rise only in the HT phase as in the case of the O(N) (N)LS [12, 13].

Now we can solve eqns. (28) for the independent vertex insertions in terms of the following operations on $\langle X\rangle$:

$$\mu\partial/\partial\mu + h\partial/\partial h - 1/2L_\sigma,\quad m\partial/\partial m,\quad u\partial/\partial u + 1/2L_\sigma,\quad f\partial/\partial f + f^*\partial/\partial f^*$$

and substitute the result into (29) to arrive at the desired set of partial differential equations for $\langle X\rangle$

$$\{\mu\partial/\partial\mu - \zeta_\Phi(f\partial/\partial f + f^*\partial/\partial f^*) + (1+2\zeta_\sigma)u\partial/\partial u + h\partial/\partial h$$

$$+\zeta_\Phi L_\Phi + \zeta_\sigma L_\sigma\}\langle X\rangle = \sum_{k=1}^{3}\omega_k\delta_{L_\sigma,k}\delta_{L_\Phi,0}\delta_{L_A,0}, \tag{30}$$

$$\{\mu\partial/\partial\mu + m\partial/\partial m + 1/2(f\partial/\partial f + f^*\partial/\partial f^*) + (1+2\gamma_\sigma)u\partial/\partial u + h\partial/\partial h$$

$$+\gamma_\Phi L_\Phi + \gamma_\sigma L_\sigma - Nm\varrho\Delta_0 - (1/2+\gamma_\Phi)\widetilde{\Delta}\}\langle X\rangle$$

$$= \sum_{k=1}^{3}\overline{\omega}_k\delta_{L_\sigma,k}\delta_{L_\Phi,0}\delta_{L_A,0}. \tag{31}$$

For the inhomogeneous terms on the r. h. s. of (30), (31) see [11]; $\overset{(-)}{\omega}_{1,2} \neq 0$ in the HT phase only. Substituting (25)—(27) into (30), (31) we obtain explicit expressions for the coefficient functions ζ_Φ, γ_Φ, ζ_σ, γ_σ (RG, CS resp., anomalous dimensions) and ϱ in terms of vertex functions $\mathscr{C}^{(L_\Phi, L_\sigma, L_A)} \equiv \langle X\rangle^{1PI}$:

$$\zeta_\Phi = \mu^2(\mu^2+m^2)^{-1}(1+\partial/\partial p^2\Gamma_{(\perp)}^{(2,0,0)})_{p^2=\mu^2},$$

$$\zeta_\sigma + 2\zeta_\Phi = 2i\mu^2(\partial/\partial\mu^2\Gamma_{(\perp)}^{(2,1,0)})_{s.p.\mu^2} - \omega_2\Gamma^{(0,2,0)}{}_{p^2=4\mu^2},$$

$$m\varrho = 2m^2[\partial/\partial p^2\Gamma^{(2,0,0)}(N\Delta_0\Gamma^{(2,0,0)})^{-1}]|_{p^2=-m^2}, \tag{32}$$

$$\gamma_\Phi - \zeta_\Phi = \frac{m^2}{m^2+\mu^2} + \left[\frac{Nm\varrho}{2(m^2+\mu^2)}\Delta_0 + (1/2+\gamma_\Phi)1/2\mu^2\widetilde{\Delta}\right]\Gamma_{(\perp)}^{(2,0,0)}|_{p^2=\mu^2},$$

$$\gamma_\sigma - \zeta_\sigma + 2(\gamma_\Phi - \zeta_\Phi) = -i[Nm\varrho\Delta_0 + (1/2+\gamma_\Phi)\widetilde{\Delta}]\Gamma_{(\perp)}^{(2,1,0)}|_{s.p.\mu^2}$$

$$-(\overline{\omega}_2 - \omega_2)\Gamma^{(0,2,0)}|_{p^2=4\mu^2}.$$

In the zero mass pre-asymptotic AH_3 theory RG and CS eqns. (30), (31) do coincide and acquire the form:

$$\{\mu\partial/\partial\mu+(1+2\eta_\sigma)u\partial/\partial u+h\partial/\partial h+\eta_\Phi L_\Phi+\eta_\sigma L_\sigma\}\langle X\rangle = \omega_3\delta_{L_\sigma,3}\delta_{L_\Phi,0}\delta_{L_A,0} \tag{30'}$$

$$\eta_\Phi(u,h)\equiv\zeta_\Phi \quad \gamma_\Phi|_{m=f=0}, \quad \eta_\sigma(u,h)\equiv\zeta_\sigma=\gamma_\sigma|_{m=f=0}.$$

From (30′) it is clear that $u^*=h^*=0$ (i. e. the critical CP_3^{N-1} theory) is an IR stable fixed point (since $\eta_\sigma=0(1/N)$ according to (32)) provided the scaling limit of the AH_3 model $u\to u^*=0$, $h\to h^*=0$ exists (cf. the Introduction). This last fact is proved in the subsequent paper [11].

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Appendix. 1/N graphical rules

The $1/N$ expansion of the generating functional for the Euclidean Green functions of models (1), (1′) (after performing the Gaussian integration over $\Phi_\perp(x)$, $\Phi^*_\perp(x)$, cf. [5, 6]):

$$Z[J,J^*,K]=\int\prod_a[d\Phi_F d\Phi^*_F d\sigma' A_\mu dB\exp\{NS_1+S_2\};$$

$$S_1=-\mathrm{Tr}\ln(-\nabla_\nu\nabla_\nu+\sigma')+\int d^3x[-(\nabla_\nu\Phi_F)^*(\nabla_\nu\Phi_F)-\sigma'(\Phi^*_F\Phi_F-\mu/T)$$
$$+u/2\mu\sigma'^2-h/4\mu F^2_{\lambda\nu}-iB\partial_\nu A_\nu],$$

$$S_2=Tr\ln(-\nabla_\nu\nabla_\nu+\sigma')+\int d^3x[K_\nu A_\nu+\sqrt{N}(J^*_F\Phi_F+\Phi^*_F J_F)]$$
$$+\int d^3x d^3y J^*_\perp(x)[-\nabla_\nu\nabla_\nu+\sigma']^{-1}(x,y)J_\perp(y),$$

$$\Phi=\Phi_\perp+\sqrt{N}\Phi_F\hat{F},\quad J=J_\perp+J_F\hat{F},\quad \sum_{a=1}^N\Phi^*_{\perp a}\hat{F}_a=\sum_{a=1}^N J^*_{\perp a}\hat{F}_a=0$$

($\hat{F}$ being an arbitrary fixed complex unit vector in "flavour" space), is generated by means of expanding around the constant saddle points of the action $S_1:\sigma'^{(c)}=\sigma_0\equiv m^2$, $\Phi_F^{(c)}\equiv f$, $A_\nu^{(c)}=B^{(c)}\equiv 0$, which are determined by exactly the same equations as for the usual $U(N)$ (N)LS model. The latter possess three types of solutions (a_0 arbitrary dimensionless non-negative constant accounting for the subtraction ambiguity in the "mass-gap" equation)

(*a*) HT phase solution: $T>T_c\equiv 4\pi(1+a_0)^{-1}$, $f=0$, $m=4\pi\mu(T_c^{-1}-T^{-1})$ for (1), $m=-\mu/8\pi u+[(\mu/8\pi u)^2+\mu^2/u(T_c^{-1}-T^{-1})]^{1/2}$ for (1′).

(*b*) LT phase solution: $T<T_c$, $m=0$, $|f|^2=\mu(T^{-1}-T_c^{-1})$.

(*c*) Critical theory (for (1)) or pre-asymptotic zero mass theory for (1′)) respectively: $T=T_c$, $m=f=0$.

The free propagators are determined from the quadratic part of NS_1+S_2 expanded around the saddle points. We write down the former in a unified "soft-mass" form, the cases (*a*), (*b*), (*c*) being distinguished by specifying the values of m, $f(m(s)=sm+(1-s)\mu)$:

$$\langle\Phi_{\perp c}\Phi^*_{\perp d}\rangle^{(0)}=[(m(s))^2+p^2]^{-1}(\delta_{cd}-\hat{F}_c\hat{F}^*_d), \tag{A.1}$$

$$\langle\Phi_F\Phi^*_F\rangle^{(0)}=\left(\sum(p^2)+u/\mu\right)(m(s)^2+p^2)^{-1}\left[\sum(p^2)+s|f|^2(m(s)^2+p^2)^{-1}+u/\mu\right]^{-1},$$

$$\langle\sigma\sigma\rangle^{(0)}=N^{-1}\left[\sum(p^2)+s|f|^2(m(s)^2+p^2)^{-1}+u/\mu\right]^{-1},$$

$$\sum(p^2)\equiv(4\pi|p|)^{-1}\,\mathrm{arctg}\,(|p|/2m(s)); \tag{A.2}$$

$$\langle A_\lambda A_\nu\rangle^{(0)}=N^{-1}(\delta_{\lambda\nu}-p_\lambda p_\nu/p^2)\mathcal{H}^{-1}(p^2), \tag{A.3}$$

$$\mathcal{H}(p^2)\equiv h/\mu p^2+(4(m(s))^2+p^2)(8\pi|p|)^{-1}\,\mathrm{arctg}\,(|p|/2m(s))-\frac{m(s)}{4\pi}+\mu(1-s)+2s|f|^2;$$

$$\langle A_\lambda B\rangle=-\langle BA_\lambda\rangle=p_\lambda/Np^2. \tag{A.4}$$

All $1/N$ graphical elements are displayed in Fig. 1 ($F\equiv\sqrt{N}f\hat{F}$). The subgraphs depicted in Fig. 2 are forbidden in diagrams of usual Green's functions. However, they do arise when extra composite operator vertex insertions of the type $\mathfrak{N}[(\Phi^*\Phi)P]$, $\mathfrak{N}[(\Phi^* i\overleftrightarrow{\partial}_\mu\Phi)P]$, etc. are present.

The particle spectrum according to (A. 1—3) consists of an $SU(N)$ N-plet of massive "flavoured" mesons and a (massless) photon in the HT phase and of only $N-1$ Goldstone bosons in the LT phase (the massive "photon" and the N^{th} Higgs meson become "confined" there).

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Обобщенные нелинейные сигма модели и универсальность в трехмерном пространстве. I. Перенормировка с переменной массой $1/N$ разложения

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(Резюме)

Построено ультрафиолетово и инфракрасно перенормированное $1/N$ разложение в обеих высоко- и низкотемпературной фазах CP^{N-1}-модели и абелевой модели Хиггса с внутренней $SU(N)$ симметрией „ароматов" в трехмерном (эвклидовом) пространстве — времени, а также в теории в самой критической точке. Доказаны обобщенные тождества квантовой киральности для составных операторов, из которых следует перенормируемость CP^{N-1}-модели. Показано, что CP^{N-1}-модель является универсальной предельной теорией для абелевой модели Хиггса в инфракрасно стабильной фиксированной точке ренормгруппы.

Phonons in the Coupled Rotational Bands Approach

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Zero-rank operators of pure phonon nature, conserving angular momentum, are proposed. They are suitable combinations inducing purely vibrational motion, of previously known elementary operators mixing vibrational with rotational degrees of freedom. Their algebraic properties have been deduced. A model space of multiphonon states is constructed with their help and their matrix elements in this space are explicitly obtained. This space is convenient for solving problems involving coupled rotational bands.

1. Introduction

The theory of coupled modes in nuclei, developed in a series of papers (see for example refs. [1—4]), is based on suitably defined transition operators — the irreducible tensors B^{+}_{aIM}[1,2], allowing to separate the kinematics from dynamics [3,4]

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О классе двумерных скалярных суперсимметричных модел с высшими квантовыми законами сохранения

Е. Р. Нисимов

(Резюме)

В работе показано, что по крайней мере в режиме слабой связи супе симметричния модель синус-Гордон единственная в классе двумерных ск лярных суперсимметричных моделей с взаимодействием без производны являющаяся вполне интегрируемой как на классическом, так и на квантово уровне. Найдено точное явное выражение для первого высшего сохраня щегося тока, откуда следует строгое доказательство отсутствия множ ственного рождения в двухчастичных столкновениях и замечательного сво ства факторизации трехчастичных процессов рассеяния, описываемых данн моделью.